DELFT UNIVERSITY OF TECHNOLOGY
I.T.S. Mathematics Departments

Quiz 1: Complex Numbers I

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The following functions can be used in this quiz:

acos, asin, atan, cos, cot, exp, ln, log, sin, sqrt, tan,

with acos=arccos, asin=arcsin, atan=arctan, cot=cotan, exp(x)=e^x and sqrt(x)=$\sqrt{x}$. Also the number e is known and one may write π = p. Multiplication is denoted by $*$ and powers use ^. For example

$$2e^{\frac{1}{3}\sin(x)} = \texttt{2*e\^{}((1/3)*sin(x))}.$$

Click on Begin Quiz to start. Answers are available after End Quiz.

Begin Quiz Answer each of the following questions.

1. If $z = \dfrac{2+i}{1-3i}$, then writing $z = \mathtt{a} + \mathtt{i} * \mathtt{b}$, with $\mathtt{a}, \mathtt{b} \in \mathbb{R}$, we have

$z =$ [ ]

2. If $w = 1 + 2i$, then $|w| =$ [ ]

3. If $u = 2e^{\frac{3}{4}\pi i}$, then writing $\overline{u} = \mathtt{a} + \mathtt{i} * \mathtt{b}$, with $\mathtt{a}, \mathtt{b} \in \mathbb{R}$, we have

$\overline{u} =$ [ ]

Correct answer:

4. Taking arg in $[0, 2\pi)$ one has

$$\arg(3-3i) = \boxed{}$$

5. Rewriting $z = \frac{(2i)^{10}}{(1+i)^{18}}$ as $z = \mathtt{a} + \mathtt{i} * \mathtt{b}$, with $\mathtt{a}, \mathtt{b} \in \mathbb{R}$, we have

$$z = \boxed{}$$

6. For all $x, y \in \mathbb{R}$ it holds that $e^{ix+y} =$

- ☐ $e^x(\sin y + i\cos y)$,
- ☐ $e^x(\cos y + i\sin y)$,
- ☐ $e^y(\sin x + i\cos x)$,
- ☐ $e^y(\cos x + i\sin x)$.

7. For all $z \in \mathbb{C}$ it holds that

- ☐ $\ln e^{|z|} = z$,
- ☐ $\ln e^{|z|} = |z|$,
- ☐ $\ln |e^z| = |z|$,
- ☐ none of the above.

Correct answer:

8. The number $w = \sqrt{2} - \sqrt{6}i$ can be written as:

$$\text{a. } w = 2\sqrt{2}\, e^{-i\frac{1}{3}\pi} \text{ and b. } w = 2\sqrt{2}\, e^{i\frac{5}{3}\pi}$$

☐ both are true, ☐ both are wrong,

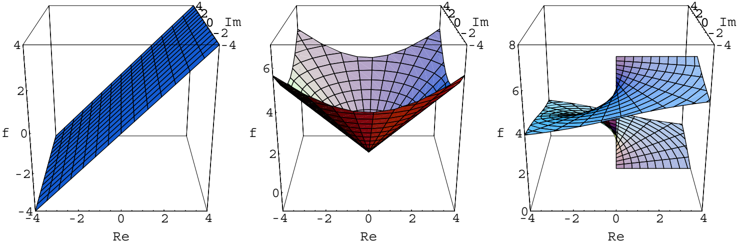
☐ a. holds, b. is wrong, ☐ b. holds, a. is wrong.

9. The number $e^{1+5i} + e^{1-5i}$ equals

☐ e^2, ☐ e^{26}, ☐ $e^2 \cos 5$,

☐ $e^{1+\ln 2} \cos 5$. ☐ none of the above.

10. The following graphs of functions $f : \mathbb{C} \to \mathbb{R}$



belong respectively to

- ☐ $f(z) = \text{Re} z$, $f(z) = \arg z$ and $f(z) = |z|$,
- ☐ $f(z) = \text{Re} z$, $f(z) = |z|$ and $f(z) = \arg z$,
- ☐ $f(z) = \arg z$, $f(z) = |z|$ and $f(z) = \text{Re} z$,
- ☐ $f(z) = |z|$, $f(z) = \arg z$ and $f(z) = \text{Re} z$.

If $\text{Im}(z^2) > 0$, then

11. $\text{Im}\, z > 0$ ☐ True ☐ False

12. $\text{Re}\, z > 0$ ☐ True ☐ False

13. $\text{Re}\, z \,.\, \text{Im}\, z > 0$ ☐ True ☐ False

14. $\text{Re}\, z > 0$ or $\text{Im}\, z > 0$

☐ True ☐ False

15. If $z \in \mathbb{C}$ is a solution of $z^4 + 2z^2 + 2 = 0$, then

$|z| =$ ☐

End Quiz | Score: | corrections

Correct answer:

After finishing the quiz one may browse through the solutions on the following pages. Also shift-click on **Ans** *jumps to the answer.*

Solutions to Quiz

Solution to Question

1. The complex conjugate of the denominator is $1 + 3i$. Multiplying both denominator and numerator by this number does not change the value but does yield a real denominator:

$$\frac{2+i}{1-3i} = \frac{(2+i)(1+3i)}{(1-3i)(1+3i)} = \frac{-1+7i}{10} = -.1 + .7i\ .$$

In the present notation:

$$-.1 + \mathtt{i} * .7\ .$$

Back to Question 1

Solution to Question

2. The modulus or absolute value of a complex number $x + iy$, with $x, y \in \mathbb{R}$ is defined by

$$|x + iy| = \sqrt{x^2 + y^2}.$$

Hence $|w| = \sqrt{1^2 + 2^2} = \sqrt{5}$. In the present notation one may use

`5^.5` or `sqrt(5)` .

Back to Question 2

Solution to Question

3. Let $x, y \in \mathbb{R}$. By definition

$$e^{x+iy} = e^x \left(\cos(y) + i\sin(y)\right).$$

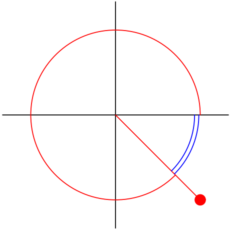
Hence $u = 2e^{\frac{3}{4}\pi i} = 2\left(\cos(\frac{3}{4}\pi) + i\sin(\frac{3}{4}\pi)\right) = -\sqrt{2} + i\sqrt{2}$ and the complex conjugate becomes

$$\overline{u} = -\sqrt{2} - i\sqrt{2}.$$

Back to Question 3

Solution to Question

4. The argument is the angle with the positive real axis measured in positive sense, that is, against the direction of the clock.



The blue angle in the picture above is $\frac{1}{4}\pi$ but since one should be measuring in clockwise direction one finds

$$\arg(3 - 3i) = 2\pi - \frac{1}{4}\pi = \frac{7}{4}\pi.$$

Back to Question 4

Solution to Question
5. One simplifies the computation by using the exponential form:

$$\frac{(2i)^{10}}{(1+i)^{18}} = \frac{\left(2e^{\frac{1}{2}\pi i}\right)^{10}}{\left(\sqrt{2}e^{\frac{1}{4}\pi i}\right)^{18}} = \frac{2^{10}\ e^{10.\frac{1}{2}\pi i}}{2^{18.\frac{1}{2}}\ e^{18.\frac{1}{4}\pi i}} = 2e^{\frac{1}{2}\pi i} = 2i\ .$$

Back to Question 5

Solution to Question

6. By definition one has for $z \in \mathbb{R}$:

$$e^z = e^{\mathrm{Re} z} \left(\cos(\mathrm{Im} z) + i \sin(\mathrm{Im} z)\right)$$

which amounts to $e^{ix+y} = e^y \left(\cos x + i \sin x\right)$. Back to Question 6

Solution to Question

7. The function ln is defined on $\mathbb{R}^+$. Since $|z|$ is real, $e^{|z|}$ is real and positive for any $z = x + iy$ with $x, y \in \mathbb{R}$ and $\ln e^{|z|} = |z|$. The other two explicit possibilities are not true in general. For example take $z = \pi i$:

$$\ln |e^{\pi i}| = \ln |i| = \ln 1 = 0,$$

which is neither equal to $|\pi i|$ nor to πi. Back to Question 7

Solution to Question

8. By straightforward computation:

$$\begin{aligned} 2\sqrt{2}\, e^{-i\frac{1}{3}\pi} &= 2\sqrt{2}\left(\cos(-\tfrac{1}{3}\pi) + i\sin(-\tfrac{1}{3}\pi)\right) \\ &= 2\sqrt{2}\left(\tfrac{1}{2} - i\tfrac{1}{2}\sqrt{3}\right) = \sqrt{2} - i\sqrt{6}. \end{aligned}$$

and

$$\begin{aligned} 2\sqrt{2}\, e^{i\frac{5}{3}\pi} &= 2\sqrt{2}\left(\cos(\tfrac{5}{3}\pi) + i\sin(\tfrac{5}{3}\pi)\right) \\ &= 2\sqrt{2}\left(\tfrac{1}{2} - i\tfrac{1}{2}\sqrt{3}\right) = \sqrt{2} - i\sqrt{6}. \end{aligned}$$

Back to Question 8

Solution to Question

9. No shortcuts. Straightforward computation yields:

$$e^{1+5i} + e^{1-5i} = e^1 \left(\cos(5) + i \sin(5)\right) + e^1 \left(\cos(5) - i \sin(5)\right) =$$
$$= 2\ e\ \cos(5) = e^{1+\ln(2)}\ \cos(5)\ .$$

Back to Question 9

Solution to Question

10. The real axis goes from left to right, the imaginary axis from front to back and the axis for the function-value from bottom to top. Since Rez is constant in the imaginary direction this should correspond with the first picture. The rotational symmetry of $|z|$ (note that $|x + iy| = \sqrt{x^2 + y^2}$) returns in the second picture.

Back to Question 10

Solution to Question
11-14. Writing $z = x + iy$ with $x, y \in \mathbb{R}$ one finds that

$$z^2 = x^2 - y^2 + 2i\,x\,y.$$

Hence $\mathrm{Im}\,\mathrm{z}^2 = 2\,\mathrm{x}\,\mathrm{y} > 0$ implies that either both x and y are positive or both are negative. Hence of these four questions only $\mathrm{Re}\,\mathrm{z}\,.\,\mathrm{Im}\,\mathrm{z} > 0$ holds true. Back to Question 11

Solution to Question 15. Since $z^4 + 2z^2 + 2 = 0$ is a quadratic polynomial in z^2 we may solve by splitting of a square:

$$z^4 + 2z^2 + 2 = \left(z^2 + 1\right)^2 + 1.$$

Hence $z^4 + 2z^2 + 2 = 0$ holds if and only if $\left(z^2 + 1\right)^2 = -1$ which means that

$$z^2 + 1 = i \text{ or } z^2 + 1 = -i.$$

In other words $z^2 = -1 \pm i$, which implies that $\left|z^2\right| = \sqrt{2}$. Hence

$$|z| = \sqrt{\sqrt{2}} = \sqrt[4]{2} = 2^{\frac{1}{4}}.$$

Back to Question 15